

On implementation issues of post-quantum cryptography

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June 13, 2019

The NIST competition

Count of Problem Catego	ry Column Labels 💌		
Row Labels	Key Exchange	Signature	Grand Total
?	1		1
Braids	1	1	2
Chebychev	1		1
Codes	19	5	24
Finite Automata	1	1	2
Hash		4	4
Hypercomplex Numbers	1		1
Isogeny	1		1
Lattice	24	4	28
Mult. Var	6	7	13
Rand. walk	1		1
RSA	1	1	2
Grand Total	57	23	80
Q 4	1] 31 ♡ 27		

Overview tweeted by Jacob Alperin-Sheriff on Dec 4, 2017.

The NIST competition (ctd.)

"Key exchange"

- What is meant is key encapsulation mechanisms (KEMs)
 - $(vk, sk) \leftarrow KeyGen()$
 - $(c, k) \leftarrow \text{Encaps}(vk)$
 - $k \leftarrow \text{Decaps}(c, \text{sk})$



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 - (vk,sk)←KeyGen()
 - (c, k)←Encaps(vk)
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Status of the NIST competition

- In total 69 submissions accepted as "complete and proper"
- Several broken, 5 withdrawn
- Jan 2019: NIST announces 26 round-2 candidates
 - 17 KEMs and PKEs
 - 9 signature schemes

NIST reference and "optimized" implementations

" Two implementations are required in the submission package: a reference implementation and an optimized implementation.

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- Allowed to use some third-party libraries:
 - NTL Version 10.5.0
 - GMP Version 6.1.2
 - OpenSSL
 - Keccak Code package

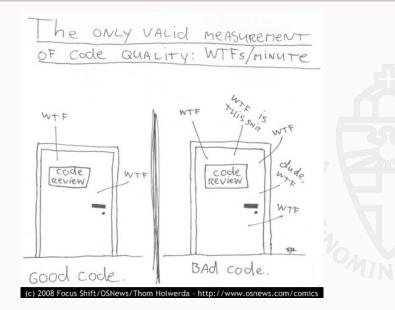
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- Allowed to use some third-party libraries:
 - NTL Version 10.5.0
 - GMP Version 6.1.2
 - OpenSSL
 - Keccak Code package
- Not allowed to use intrinsics or assembly
- Can include additional (e.g., architecture-specific) implementations

Code quality



- Joint work with Matthias Kannwischer, Joost Rijneveld, Douglas Stebila, Thom Wiggers
- GitHub repo with extensive CI to ensure "clean" implementations

PQClean

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- Goal: collect "clean C" code of all round-2 candidates
- Make it easy to use in other projects
- Make it easy to use as starting point for optimization

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- Goal: collect "clean C" code of all round-2 candidates
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- Make it easy to use as starting point for optimization
- Longer-term, if there is interest:
 - implementations with architecture-specific optimizations?
 - implementations in other languages?

- Code is valid C99
- Passes functional tests
- API functions do not write outside provided buffers
- API functions do not need pointers to be aligned
- Compiles with -Wall -Wextra -Wpedantic -Werror with gcc and clang
- Compiles with /W4 /WX with MS compiler
- Consistent test vectors across runs
- Consistent test vectors on big-endian and little-endian machines
- Consistent test vectors on 32-bit and 64-bit machines

- No errors/warnings reported by valgrind
- No errors/warnings reported by address sanitizer
- No errors/warnings reported by undefined-behavior sanitizer
- Only dependencies:
 - fips202.c
 - sha2.c
 - aes.c
 - randombytes.c

- API functions return 0 on success, negative on failure
- No dynamic memory allocations



- API functions return 0 on success, negative on failure
- No dynamic memory allocations
- Builds under Linux, MacOS, and Windows without warnings
- All exported symbols are namespaced with PQCLEAN_SCHEMENAME_
- Each implementation comes with license and meta information in META.yml

• No variable-length arrays (required to build under Windows)



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- Argument names consistent between .h and .c files

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- Valgrind does not work with environments running on qemu

CRYSTALS-Dilithium	✓
FALCON	
GeMSS	
LUOV	WIP
MQDSS	✓
Picnic	
qTESLA	—
Rainbow	WIP
SPHINCS+	✓



PQClean status quo – KEMs

BIKE	
Classic McEliece	WIP
CRYSTALS-Kyber	1
Frodo-KEM	1
HQC	
LAC	
LEDAcrypt	WIP
NewHope	1
NTRU	1
NTRU Prime	WIP
NTS-KEM	
ROLLO	
Round5	
RQC	
SABER	
SIKE	
ThreeBears	WIP



• Copy files from origin directory



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- Instantiate SHA-3, SHA-2, AES (or copy from PQClean)

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- Instantiate SHA-3, SHA-2, AES (or copy from PQClean)
- Add .c and .h files to build system

• Joint work with

Matthias Kannwischer, Joost Rijneveld, and Ko Stoffelen.

- Started as part of PQCRYPTO H2020 project
- Continued within EPOQUE ERC StG
- Library and testing/benchmarking framework
 - PQ-crypto on ARM Cortex-M4
 - Uses STM32F4 Discovery board
 - 192 KB of RAM, benchmarks at 24 MHz
- Easy to add schemes using NIST API
- Optimized SHA3 and AES shared across primitives



 Run functional tests of all primitives and implementations: python3 test.py



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- Generate testvectors, compare for consistency (also with host): python3 testvectors.py
- Run speed and stack benchmarks: python3 benchmarks.py
- Easy to evaluate only subset of schemes, e.g.: python3 test.py newhope1024cca sphincs-shake256-128s

CRYSTALS-Dilithium FALCON GeMSS LUOV MQDSS Picnic qTESLA Rainbow SPHINCS+

X

X

X



KEMs (not) in pqm4

	ref/clean	opt	
BIKE	—	—	
Classic McEliece	×	×	
CRYSTALS-Kyber	\checkmark	\checkmark	
Frodo-KEM	\checkmark	(🗸)	
HQC	—		
LAC	\checkmark	—	
LEDAcrypt	WIP	WIP	1.90
NewHope	\checkmark	\checkmark	
NTRU	\checkmark	\checkmark	
NTRU Prime	\checkmark	—	
NTS-KEM	×	×	
ROLLO	—	—	
Round5	WIP	WIP	
RQC	_		MING
SABER	\checkmark	\checkmark	
SIKE	—	—	
ThreeBears	\checkmark	1	15

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BIKE	—	—	
Classic McEliece	×	X	
CRYSTALS-Kyber	\checkmark	\checkmark	
Frodo-KEM	\checkmark	(✔)	
HQC	—	—	
LAC	\checkmark	—	
LEDAcrypt	WIP	WIP	
NewHope	\checkmark	\checkmark	
NTRU	\checkmark	\checkmark	
NTRU Prime	\checkmark	_	
NTS-KEM	×	X	
ROLLO	—	—	
Round5	WIP	WIP	
RQC	_	_	MINE
SABER	1	\checkmark	
SIKE	_	_	
ThreeBears	1	\checkmark	15

- Given uniform $\mathbf{A} \in \mathbb{Z}_q^{k imes \ell}$
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Alice (server)		Bob (client)
$\mathbf{s}, \mathbf{e} \xleftarrow{\hspace{0.15cm}} \chi$		$\mathbf{s'}, \mathbf{e'} \xleftarrow{\hspace{0.15cm} \$} \chi$
$\mathbf{b} \leftarrow \mathbf{as} + \mathbf{e}$	$\xrightarrow{ \ \ b} \\$	$\mathbf{u}{\leftarrow}\mathbf{a}\mathbf{s}'+\mathbf{e}'$
	←	

Alice has $\mathbf{v} = \mathbf{us} = \mathbf{ass'} + \mathbf{e's}$ Bob has $\mathbf{v'} = \mathbf{bs'} = \mathbf{ass'} + \mathbf{es'}$

- Secret and noise $\boldsymbol{s}, \boldsymbol{s}', \boldsymbol{e}, \boldsymbol{e}'$ are small
- **v** and **v**' are *approximately* the same

Core operation: multiplication in $\mathcal{R}_q = \mathbb{Z}_q[X]/f$

Power-of-two q

- Several schemes use $q = 2^m$, for small m
- Examples: Round5, NTRU, Saber
- More round-1 examples: Kindi, RLizard



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Prime "NTT-friendly" q

- Kyber and NewHope use prime q supporting fast NTT
- For $A, B \in \mathcal{R}_q$, $A \cdot B = \mathsf{NTT}^{-1}(\mathsf{NTT}(A) \circ \mathsf{NTT}(B))$
- NTT is Fourier Transform over finite field
- Use $f = X^n + 1$ for power-of-two n

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- Can do better using Karatsuba:

$$(a_{\ell} + X^{k}a_{h}) \cdot (b_{\ell} + X^{k}b_{h})$$

= $a_{\ell}b_{\ell} + X^{k}(a_{\ell}b_{h} + a_{h}b_{\ell}) + X^{n}a_{h}b_{h}$
= $a_{\ell}b_{\ell} + X^{k}((a_{\ell} + a_{h})(b_{\ell} + b_{h}) - a_{\ell}b_{\ell} - a_{h}b_{h}) + X^{n}a_{h}b_{h}$

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- Generalization: Toom-Cook
 - Toom-3: split into 5 multiplications of 1/3 size
 - Toom-4: split into 7 multiplications of 1/4 size
- Approach: Evaluate, multiply, interpolate

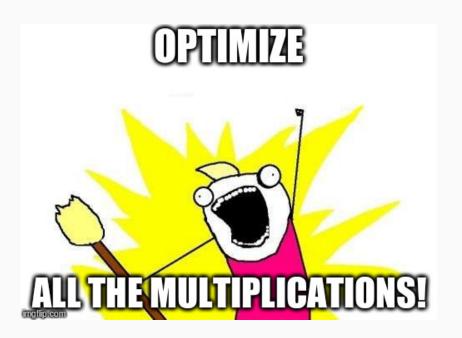
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- Is this the best approach? How about other values of q and n?



- Generate optimized assembly for Karatsuba/Toom
- Use Python scripts, receive as input n and q
- Hand-optimize "small" schoolbook multiplications
 - Make heavy use of "vector instructions"
 - Perform two 16 \times 16-bit multiply-accumulate in one cycle
 - Carefully schedule instructions to minimize loads/stores
- Benchmark different options, pick fastest

Multiplication results

	approach	"small"	cycles	stack
	Karatsuba only	16	41 121	2 0 2 0
Saber	Toom-3	11	41 225	3 480
$(n = 256, q = 2^{13})$	Toom-4	16	39 124	3 800
	Toom-4 + Toom-3	-	-	-
	Karatsuba only	16	41 121	2 0 2 0
Kindi-256-3-4-2	Toom-3	11	41 225	3 480
$(n = 256, q = 2^{14})$	Toom-4	-	-	_
	Toom-4 + Toom-3	-	-	- (
	Karatsuba only	11	230 132	5 676
NTRU-HRSS	Toom-3	15	217 436	9 384
$(n = 701, q = 2^{13})$	Toom-4	11	182 129	10 596
	Toom-4 + Toom-3	-		
	Karatsuba only	12	247 489	6 0 1 2
NTRU-KEM-743	Toom-3	16	219 061	9 920
$(n = 743, q = 2^{11})$	Toom-4	12	196 940	11 208
	Toom-4 + Toom-3	16	197 227	12 152
RLizard-1024	Karatsuba only	16	400 810	8 1 8 8
	Toom-3	11	360 589	13756
$(n = 1024, \dots, 211)$	Toom-4	16	313 744	15 344
$q = 2^{11})$	Toom-4 + Toom-3	11	315 788	16816

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- Primary goal: optimize Kyber
- Secondary effect: optimize NewHope (with room for improvement)

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- NTT is an FFT in a finite field
- Evaluate polynomial $f = f_0 + f_1 X + \dots + f_{n-1} X^{n-1}$ at all *n*-th roots of unity
- Divide-and-conquer approach
 - Write polynomial f as $f_0(X^2) + X f_1(X^2)$

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$$f(\beta) = f_0(\beta^2) + \beta f_1(\beta^2) \text{ and}$$
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- Primary goal: optimize Kyber
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- NTT is an FFT in a finite field
- Evaluate polynomial f = f₀ + f₁X + ··· + f_{n-1}Xⁿ⁻¹ at all n-th roots of unity
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$$f(\beta) = f_0(\beta^2) + \beta f_1(\beta^2) \text{ and}$$
$$f(-\beta) = f_0(\beta^2) - \beta f_1(\beta^2)$$

- f₀ has n/2 coefficients
- Evaluate f_0 at all (n/2)-th roots of unity by recursive application
- Same for f₁

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- Loop over log n levels with n/2 "butterflies" each



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- Loop over log n levels with n/2 "butterflies" each
- Butterfly on level k:
 - Pick up f_i and f_{i+2^k}
 - Multiply f_{i+2^k} by a power of ω to obtain t
 - Compute $f_{i+2^k} \leftarrow a_i t$
 - Compute $f_i \leftarrow a_i + t$



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 - Compute $f_i \leftarrow a_i + t$
- Main optimizations on Cortex-M4:
 - "Merge" levels: fewer loads/stores
 - Optimize modular arithmetic (precompute powers of ω in Montgomery domain)
 - Lazy reductions
 - Carefully optimize using DSP instructions

Optimized lattice KEM cycles

Scheme	Key Generation	Encapsulation	Decapsulation
ntruhps2048509	77 698 713	645 329	542 439
ntruhps2048677	144 383 491	955 902	836 959
ntruhps4096821	211 758 452	1 205 662	1 066 879
ntruhrss701	154 676 705	402 784	890 231
lightsaber	459 965	651 273	678 810
saber	896 035	1 161 849	1 204 633
firesaber	1 448 776	1 786 930	1 853 339
kyber512	514 291	652769	621 245
kyber768	976 757	1 146 556	1 094 849
kyber1024	1 575 052	1 779 848	1 709 348
newhope1024cpa	1 034 955	1 495 457	206 112
newhope1024cca	1 219 908	1 903 231	1 927 505

Comparison: Curve25519 scalarmult: 625358 cycles

Optimized lattice KEM stack bytes

Scheme	Key Generation	Encapsulation	Decapsulation
ntruhps2048509	21 412	15 452	14 828
ntruhps2048677	28 524	20 604	19756
ntruhps4096821	34 532	24 924	23 980
ntruhrss701	27 580	19 372	20 580
lightsaber	9 656	11 392	12 136
saber	13 256	15 544	16640
firesaber	20 144	23 008	24 592
kyber512	2 952	2 552	2 560
kyber768	3 848	3 1 2 8	3072
kyber1024	4 360	3 584	3 592
newhope1024cpa	11 128	17 288	8 328
newhope1024cca	11 152	17 400	19 640

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- Long-term solution: hardware acceleration for Keccak
- Much more work to be done on code-based KEMs
- So far very little work on SCA protection
- Start with "constant-time" software for all candidates
- Formally verify constant-time behavior? Definition?
- Would be great to have hacspec implementations of all NIST candidates

- PQClean repository: https://github.com/PQClean/PQClean
- pqm4 library and benchmarking suite: https://github.com/mupq/pqm4
- pqriscv library and benchmarking suite: https://github.com/mupq/pqriscv
- Code of $\mathbb{Z}_{2^m}[x]$ multiplication paper, including scripts: https://github.com/mupq/polymul-z2mx-m4
- Z_{2^m}[x] multiplication paper: https://cryptojedi.org/papers/#latticem4
- Kyber optimization paper: https://cryptojedi.org/papers/#nttm4