## Radboud University

## On implementation issues of post-quantum cryptography

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| Count of Problem Category Column Labels $\boldsymbol{\nabla}$ |  |  |  |
| :---: | :---: | :---: | :---: |
| Row Labels | - Key Exchange | Signature | Grand Total |
| ? | 1 |  | 1 |
| Braids | 1 | 1 | 2 |
| Chebychev | 1 |  | 1 |
| Codes | 19 | 5 | 24 |
| Finite Automata | 1 | 1 | 2 |
| Hash |  | 4 | 4 |
| Hypercomplex Numbers | 1 |  | 1 |
| Isogeny | 1 |  | 1 |
| Lattice | 24 | 4 | 28 |
| Mult. Var | 6 | 7 | 13 |
| Rand. walk | 1 |  | 1 |
| RSA | 1 | 1 | 2 |
| Grand Total | 57 | 23 | 80 |
| Q 4 | 饣ᄀ31 $\mathrm{O}_{27}$ | $\bullet$ |  |

Overview tweeted by Jacob Alperin-Sheriff on Dec 4, 2017.

## The NIST competition (ctd.)

"Key exchange"

- What is meant is key encapsulation mechanisms (KEMs)
- ( $\mathrm{vk}, \mathrm{sk}) \leftarrow \operatorname{KeyGen}()$
- $(c, k) \leftarrow E n c a p s(v k)$
- $k \leftarrow \operatorname{Decaps}(c, s k)$


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## Status of the NIST competition

- In total 69 submissions accepted as "complete and proper"
- Several broken, 5 withdrawn
- Jan 2019: NIST announces 26 round-2 candidates
- 17 KEMs and PKEs
- 9 signature schemes


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- NTL Version 10.5.0
- GMP Version 6.1.2
- OpenSSL
- Keccak Code package


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- Allowed to use some third-party libraries:
- NTL Version 10.5.0
- GMP Version 6.1.2
- OpenSSL
- Keccak Code package
- Not allowed to use intrinsics or assembly
- Can include additional (e.g., architecture-specific) implementations

Code quality
The oncy vacid measurement of code quality: WTFs/minute


- Joint work with

Matthias Kannwischer, Joost Rijneveld, Douglas Stebila, Thom Wiggers

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- Make it easy to use in other projects
- Make it easy to use as starting point for optimization
- Longer-term, if there is interest:
- implementations with architecture-specific optimizations?
- implementations in other languages?
- Code is valid C99
- Passes functional tests
- API functions do not write outside provided buffers
- API functions do not need pointers to be aligned
- Compiles with -Wall -Wextra -Wpedantic -Werror with gcc and clang
- Compiles with /W4 /WX with MS compiler
- Consistent test vectors across runs
- Consistent test vectors on big-endian and little-endian machines
- Consistent test vectors on 32 -bit and 64 -bit machines
- No errors/warnings reported by valgrind
- No errors/warnings reported by address sanitizer
- No errors/warnings reported by undefined-behavior sanitizer
- Only dependencies:
- fips202.c
- sha2.c
- aes.c
- randombytes.c
- API functions return 0 on success, negative on failure
- No dynamic memory allocations
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- Builds under Linux, MacOS, and Windows without warnings
- All exported symbols are namespaced with PQCLEAN_SCHEMENAME_
- Each implementation comes with license and meta information in META.yml


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- Dealing with controversial warnings (unary minus on unsigned integers)
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- Separate subdirectories (without symlinks) for each parameter set of each scheme
- \#ifdefs only for header encapsulation
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- Dealing with controversial warnings (unary minus on unsigned integers)
- Argument names consistent between .h and .c files


## Limitations and lessons learned

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- Could use valgrind with uninitialized secret data (dynamic)
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- Valgrind does not work with environments running on qemu

| CRYSTALS-Dilithium | $\checkmark$ |
| :--- | :--- |
| FALCON | - |
| GeMSS | - |
| LUOV | WIP |
| MQDSS | $\checkmark$ |
| Picnic | - |
| qTESLA | - |
| Rainbow | WIP |
| SPHINCS+ | $\checkmark$ |

PQClean status quo - KEMs

| BIKE | - |
| :--- | :--- |
| Classic McEliece | WIP |
| CRYSTALS-Kyber | $\checkmark$ |
| Frodo-KEM | $\checkmark$ |
| HQC | - |
| LAC | - |
| LEDAcrypt | WIP |
| NewHope | $\checkmark$ |
| NTRU | $\checkmark$ |
| NTRU Prime | WIP |
| NTS-KEM | - |
| ROLLO | - |
| Round5 | - |
| RQC | - |
| SABER | - |
| SIKE | - |
| ThreeBears | WIP |

## Using code from PQClean

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- Instantiate SHA-3, SHA-2, AES (or copy from PQClean)
- Add .c and .h files to build system
- Joint work with

Matthias Kannwischer, Joost Rijneveld, and Ko Stoffelen.

- Started as part of PQCRYPTO H2020 project
- Continued within EPOQUE ERC StG
- Library and testing/benchmarking framework
- PQ-crypto on ARM Cortex-M4
- Uses STM32F4 Discovery board
- 192 KB of RAM, benchmarks at 24 MHz
- Easy to add schemes using NIST API
- Optimized SHA3 and AES shared across primitives


## pqm4 usage

- Run functional tests of all primitives and implementations: python3 test.py
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- Generate testvectors, compare for consistency (also with host): python3 testvectors.py
- Run speed and stack benchmarks:
python3 benchmarks.py
- Easy to evaluate only subset of schemes, e.g.: python3 test.py newhope1024cca sphincs-shake256-128s


## Signatures (not) in pqm4



## KEMs (not) in pqm4

|  | ref/clean | opt |
| :--- | :--- | :--- |
| BIKE | - | - |
| Classic McEliece | $x$ | $x$ |
| CRYSTALS-Kyber | $\checkmark$ | $\checkmark$ |
| Frodo-KEM | $\checkmark$ | $(\checkmark)$ |
| HQC | - | - |
| LAC | $\checkmark$ | - |
| LEDAcrypt | WIP | WIP |
| NewHope | $\checkmark$ | $\checkmark$ |
| NTRU | $\checkmark$ | $\checkmark$ |
| NTRU Prime | $\boxed{ }$ | - |
| NTS-KEM | - | - |
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## Learning with errors (LWE)

- Given uniform $\mathbf{A} \in \mathbb{Z}_{q}^{k \times \ell}$
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- Structured lattices: work in $\mathbb{Z}_{q}[x] / f$


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| Alice (server) |  | Bob (client) |
| :--- | :--- | :--- |
| $\mathbf{s}, \mathbf{e}{ }^{s} \chi$ |  | $\mathbf{s}^{\prime}, \mathbf{e}^{\prime} \leftarrow^{s} \chi$ |
| $\mathbf{b} \leftarrow \mathbf{a s}+\mathbf{e}$ | $\underset{\mathbf{b}}{\leftrightarrows}$ | $\mathbf{u} \leftarrow \mathbf{a s}^{\prime}+\mathbf{e}^{\prime}$ |
|  | $\longleftarrow$ |  |

Alice has $\mathbf{v}=\mathbf{u s}=$ ass $^{\prime}+\mathbf{e}^{\prime} \mathbf{s}$
Bob has $\mathbf{v}^{\prime}=\mathbf{b s}^{\prime}=\mathbf{a s s}^{\prime}+\mathbf{e s}^{\prime}$

- Secret and noise $\mathbf{s}, \mathbf{s}^{\prime}, \mathbf{e}, \mathbf{e}^{\prime}$ are small
- $\mathbf{v}$ and $\mathbf{v}^{\prime}$ are approximately the same


## Core operation: multiplication in $\mathcal{R}_{q}=\mathbb{Z}_{q}[X] / f$

Power-of-two q

- Several schemes use $q=2^{m}$, for small $m$
- Examples: Round5, NTRU, Saber
- More round-1 examples: Kindi, RLizard


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## Prime "NTT-friendly" $q$

- Kyber and NewHope use prime $q$ supporting fast NTT
- For $A, B \in \mathcal{R}_{q}, A \cdot B=\mathrm{NTT}^{-1}(\mathrm{NTT}(A) \circ \mathrm{NTT}(B))$
- NTT is Fourier Transform over finite field
- Use $f=X^{n}+1$ for power-of-two $n$


## Multiplication in $\mathbb{Z}_{2^{m}}[X]$

- Joint work with Matthias Kannwischer and Joost Rijneveld
- Represent coefficients as 16 -bit integers
- No modular reductions required, $2^{16}$ is a multiple of $q=2^{m}$


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$$
\begin{aligned}
& \left(a_{\ell}+X^{k} a_{h}\right) \cdot\left(b_{\ell}+X^{k} b_{h}\right) \\
= & a_{\ell} b_{\ell}+X^{k}\left(a_{\ell} b_{h}+a_{h} b_{\ell}\right)+X^{n} a_{h} b_{h} \\
= & a_{\ell} b_{\ell}+X^{k}\left(\left(a_{\ell}+a_{h}\right)\left(b_{\ell}+b_{h}\right)-a_{\ell} b_{\ell}-a_{h} b_{h}\right)+X^{n} a_{h} b_{h}
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- Recursive application yields complexity $\Theta\left(n^{\log _{2} 3}\right)$
- Generalization: Toom-Cook
- Toom-3: split into 5 multiplications of $1 / 3$ size
- Toom-4: split into 7 multiplications of $1 / 4$ size
- Approach: Evaluate, multiply, interpolate


## Initial observations

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- Karmakar, Bermudo Mera, Sinha Roy, Verbauwhede (CHES 2018):
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- Is this the best approach? How about other values of $q$ and $n$ ?


## 



- Generate optimized assembly for Karatsuba/Toom
- Use Python scripts, receive as input $n$ and $q$
- Hand-optimize "small" schoolbook multiplications
- Make heavy use of "vector instructions"
- Perform two $16 \times 16$-bit multiply-accumulate in one cycle
- Carefully schedule instructions to minimize loads/stores
- Benchmark different options, pick fastest


## Multiplication results

|  | approach | "small" | cycles | stack |
| :---: | :---: | :---: | :---: | :---: |
| Saber$\left(n=256, q=2^{13}\right)$ | Karatsuba only | 16 | 41121 | 2020 |
|  | Toom-3 | 11 | 41225 | 3480 |
|  | Toom-4 | 16 | 39124 | 3800 |
|  | Toom-4 + Toom-3 | - | - | - |
| $\begin{aligned} & \text { Kindi-256-3-4-2 } \\ & \left(n=256, q=2^{14}\right) \end{aligned}$ | Karatsuba only | 16 | 41121 | 2020 |
|  | Toom-3 | 11 | 41225 | 3480 |
|  | Toom-4 | - | - | - |
|  | Toom-4 + Toom-3 | - | - | - |
| NTRU-HRSS$\left(n=701, q=2^{13}\right)$ | Karatsuba only | 11 | 230132 | 5676 |
|  | Toom-3 | 15 | 217436 | 9384 |
|  | Toom-4 | 11 | 182129 | 10596 |
|  | Toom-4 + Toom-3 | - | - | - |
| NTRU-KEM-743$\left(n=743, q=2^{11}\right)$ | Karatsuba only | 12 | 247489 | 6012 |
|  | Toom-3 | 16 | 219061 | 9920 |
|  | Toom-4 | 12 | 196940 | 11208 |
|  | Toom-4 + Toom-3 | 16 | 197227 | 12152 |
| $\begin{aligned} & \text { RLizard-1024 } \\ & (n=1024, \\ & \left.q=2^{11}\right) \end{aligned}$ | Karatsuba only | 16 | 400810 | 8188 |
|  | Toom-3 | 11 | 360589 | 13756 |
|  | Toom-4 | 16 | 313744 | $15344$ |
|  | Toom-4 + Toom-3 | 11 | 315788 | 16816 |

## NTT-based multiplication

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- Primary goal: optimize Kyber
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- Evaluate polynomial $f=f_{0}+f_{1} X+\cdots+f_{n-1} X^{n-1}$ at all $n$-th roots of unity
- Divide-and-conquer approach
- Write polynomial $f$ as $f_{0}\left(X^{2}\right)+X f_{1}\left(X^{2}\right)$


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$$
\begin{aligned}
f(\beta) & =f_{0}\left(\beta^{2}\right)+\beta f_{1}\left(\beta^{2}\right) \text { and } \\
f(-\beta) & =f_{0}\left(\beta^{2}\right)-\beta f_{1}\left(\beta^{2}\right)
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- $f_{0}$ has $n / 2$ coefficients
- Evaluate $f_{0}$ at all ( $n / 2$ )-th roots of unity by recursive application
- Same for $f_{1}$


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- Loop over $\log n$ levels with $n / 2$ "butterflies" each
- Butterfly on level $k$ :
- Pick up $f_{i}$ and $f_{i+2^{k}}$
- Multiply $f_{i+2^{k}}$ by a power of $\omega$ to obtain $t$
- Compute $f_{i+2^{k}} \leftarrow a_{i}-t$
- Compute $f_{i} \leftarrow a_{i}+t$


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- Compute $f_{i+2^{k}} \leftarrow a_{i}-t$
- Compute $f_{i} \leftarrow a_{i}+t$
- Main optimizations on Cortex-M4:
- "Merge" levels: fewer loads/stores
- Optimize modular arithmetic (precompute powers of $\omega$ in Montgomery domain)
- Lazy reductions
- Carefully optimize using DSP instructions


## Optimized lattice KEM cycles

| Scheme | Key Generation | Encapsulation | Decapsulation |
| :--- | ---: | ---: | ---: |
| ntruhps2048509 | 77698713 | 645329 | 542439 |
| ntruhps2048677 | 144383491 | 955902 | 836959 |
| ntruhps4096821 | 211758452 | 1205662 | 1066879 |
| ntruhrss701 | 154676705 | 402784 | 890231 |
| lightsaber | 459965 | 651273 | 678810 |
| saber | 896035 | 1161849 | 1204633 |
| firesaber | 1448776 | 1786930 | 1853339 |
| kyber512 | 514291 | 652769 | 621245 |
| kyber768 | 976757 | 1146556 | 1094849 |
| kyber1024 | 1575052 | 1779848 | 1709348 |
| newhope1024cpa | 1034955 | 1495457 | 206112 |
| newhope1024cca | 1219908 | 1903231 | 1927505 |

Comparison: Curve25519 scalarmult: 625358 cycles

## Optimized lattice KEM stack bytes

| Scheme | Key Generation | Encapsulation | Decapsulation |
| :--- | ---: | ---: | ---: |
| ntruhps2048509 | 21412 | 15452 | 14828 |
| ntruhps2048677 | 28524 | 20604 | 19756 |
| ntruhps4096821 | 34532 | 24924 | 23980 |
| ntruhrss701 | 27580 | 19372 | 20580 |
| lightsaber | 9656 | 11392 | 12136 |
| saber | 13256 | 15544 | 16640 |
| firesaber | 20144 | 23008 | 24592 |
| kyber512 | 2952 | 2552 | 2560 |
| kyber768 | 3848 | 3128 | 3072 |
| kyber1024 | 4360 | 3584 | 3592 |
| newhope1024cpa | 11128 | 17288 | 8328 |
| newhope1024cca | 11152 | 17400 | 19640 |

## Conclusions and open questions

- Speed-bottleneck of lattice-based KEMs is Keccak
- Long-term solution: hardware acceleration for Keccak


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- Long-term solution: hardware acceleration for Keccak
- Much more work to be done on code-based KEMs
- So far very little work on SCA protection
- Start with "constant-time" software for all candidates


## Conclusions and open questions

- Speed-bottleneck of lattice-based KEMs is Keccak
- Long-term solution: hardware acceleration for Keccak
- Much more work to be done on code-based KEMs
- So far very little work on SCA protection
- Start with "constant-time" software for all candidates
- Formally verify constant-time behavior? Definition?
- Would be great to have hacspec implementations of all NIST candidates
- PQClean repository: https://github.com/PQClean/PQClean
- pqm4 library and benchmarking suite: https://github.com/mupq/pqm4
- pqriscv library and benchmarking suite: https://github.com/mupq/pqriscv
- Code of $\mathbb{Z}_{2^{m}}[x]$ multiplication paper, including scripts: https://github.com/mupq/polymul-z2mx-m4
- $\mathbb{Z}_{2^{m}}[x]$ multiplication paper: https://cryptojedi.org/papers/\#latticem4
- Kyber optimization paper:
https://cryptojedi.org/papers/\#nttm4

