High-speed high-security signatures

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- Elliptic-curve signature scheme and corresponding software
- Based on arithmetic on twisted Edwards curves

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Speed features

- Fast signing: 87548 cycles on Intel Nehalem/Westmere
- Fast verification: 273364 cycles
- Even faster batch verification: < 134000 cycles/signature
- Fast key generation: 93288 cycles
- Short signatures (64 bytes), short public keys (32 bytes)

Recall Schnorr signatures

- Variant of ElGamal Signatures
- Many more variants (DSA, ECDSA, KCDSA, ...)
- Uses finite group $G = \langle B \rangle$, with $|G| = \ell$
- Uses hash-function $H : G \times \mathbb{Z} \to \{0, \ldots, 2^t - 1\}$
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- Verifier computes $\overline{R} = SB + H(R, M)A$ and checks that

$$H(\overline{R}, M) = H(R, M)$$

EdDSA and Ed25519 parameters

EdDSA

- Integer $b \geq 10$

Ed25519-SHA-512

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- $B \in \{(x, y) \in \mathbb{F}_q \times \mathbb{F}_q, -x^2 + y^2 = 1 + dx^2y^2\}$ (twisted Edwards curve E)
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Ed25519 curve is birationally equivalent to the Curve25519 curve.

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- Compute A from $\underline{A}$: $x_A = \pm\sqrt{(y_A^2 - 1)/(dy_A^2 + 1)}$

EdDSA signatures

Signing

- Message M determines $r = H(h_b, \ldots, h_{2b-1}, M) \in \{0, \ldots, 2^{2b} - 1\}$
- Define $R = rB$
- Define $S = (r + H(\underline{R}, \underline{A}, M)a) \mod \ell$
- Signature: $(\underline{R}, \underline{S})$, with $\underline{S}$ the b-bit little-endian encoding of S
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Verification

- Verifier parses A from $\underline{A}$ and R from $\underline{R}$
- Computes $H(\underline{R}, \underline{A}, M)$
- Checks group equation

$$8SB = 8R + 8H(\underline{R}, \underline{A}, M)A$$

- Rejects if parsing fails or equation does not hold

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- Signatures are hash-function-collision resilient
- Including $\underline{A}$ alleviates concerns about attacks against multiple keys

Foolproof session keys

- Each message needs a different, hard-to-predict r (“session key”)
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- Same security as random r under standard PRF assumptions
- Does not consume per-message randomness
- Better for testing (deterministic output)

Fast arithmetic in $\mathbb{F}_{2^{255}-19}$

Radix 2^{64}

- Standard: break elements of $\mathbb{F}_{2^{255}-19}$ into 4 64-bit integers
- (Schoolbook) multiplication breaks down into 16 64-bit integer multiplications
- Adding up partial results requires many add-with-carry (`adc`)
- Westmere bottleneck: 1 `adc` every two cycles vs. 3 `add` per cycle

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Radix 2^{51}

- Instead break into 5 64-bit integers, use radix 2^{51}
- Schoolbook multiplication now 25 64-bit integer multiplications
- Partial results have < 128 bits, adding upper part is `add`, not `adc`
- Easy to merge multiplication with reduction (multiplies by 19)
- Better performance on Westmere/Nehalem, worse on 65 nm Core 2 and AMD processors

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- Signing takes 87548 cycles on an Intel Westmere CPU
- Key generation takes about 6000 cycles more (read from `/dev/urandom`)

Fast verification

- First part: point decompression, compute x coordinate x_R of R as

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- Use Bos-Coster algorithm for multi-scalar multiplication
- Verifying a batch of 64 signatures takes 8.55 million cycles (i.e., < 134000 cycles/signature)

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- Typical heap root replacement (pop operation): start at the root, swap down for a variable amount of times
- Floyd's heap: swap down to the bottom, swap up for a variable amount of times, advantages:
  - Each swap-down step needs only one comparison (instead of two)
  - Swap-down loop is more friendly to branch predictors

Results

- New fast and secure signature scheme
- (Slow) C and Python reference implementations
- Fast AMD64 assembly implementations
- Also new speed records for Curve25519 ECDH
- All software in the public domain and included in eBATS
- All reported benchmarks (except batch verification) are eBATS benchmarks
- All reported benchmarks had TurboBoost switched off
- Software to be included in the NaCl library

http://ed25519.cr.yp.to/