MAX PLANCK INSTITUTE
FOR SECURITY AND PRIVACY

Kyber – Design

July 31, 2024

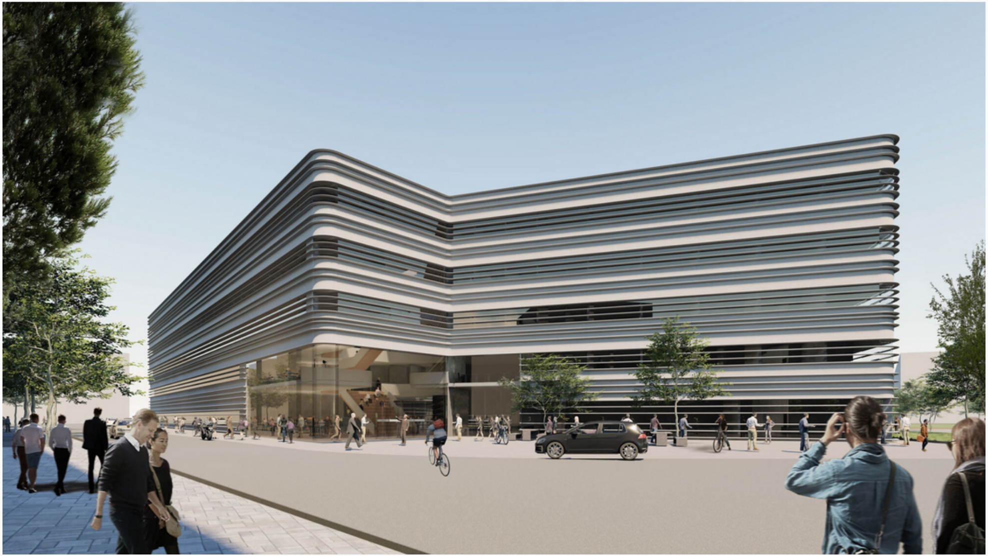
> *“It’s going to be exhausting. You will start feeling it around the middle of the week.”*
>
> *—Peter Druschel, July 29, 2024*

MPI-SP?

- Located in Bochum
- Founded in 2019
- Currently 11 PIs
- Aim to have
  - 6 Directors
  - 12 MPRGLs
  - Around 250 people total
- Currently on RUB campus

From `https://imprs-brain-behavior.mpg.de/Max_Planck_Society`

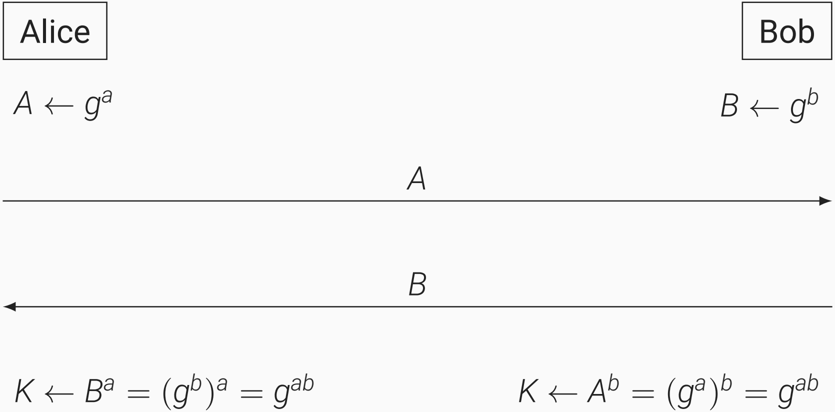
Our new home (move planned for 2027)



[A small demo]

ECDH and X25519

Let G be a finite cyclic group with generator g.



Alice

Bob

$A \leftarrow g^a$

$B \leftarrow g^b$

$$\xrightarrow{\quad A \quad}$$

$$\xleftarrow{\quad B \quad}$$

$K \leftarrow B^a = (g^b)^a = g^{ab}$

$K \leftarrow A^b = (g^a)^b = g^{ab}$

ECDH and X25519

- Diffie, Hellman, 1976: Use $G = GF(q)^*$
- Miller, Koblitz (independently), 1985/86: Use group of points on an elliptic curve
- Bernstein, 2006: Use specific elliptic curve over $GF(2^{255} - 19)$

(EC)DH is everywhere

Nitrokey
secure your digital life

WIREGUARD
FAST, MODERN, SECURE VPN TUNNEL

Tor

zoom

The Discrete Logarithm Problem

Definition

Given $P, Q \in G$ such that $Q \in \langle P \rangle$, find an integer k such that $P^k = Q$.

The Discrete Logarithm Problem

Definition

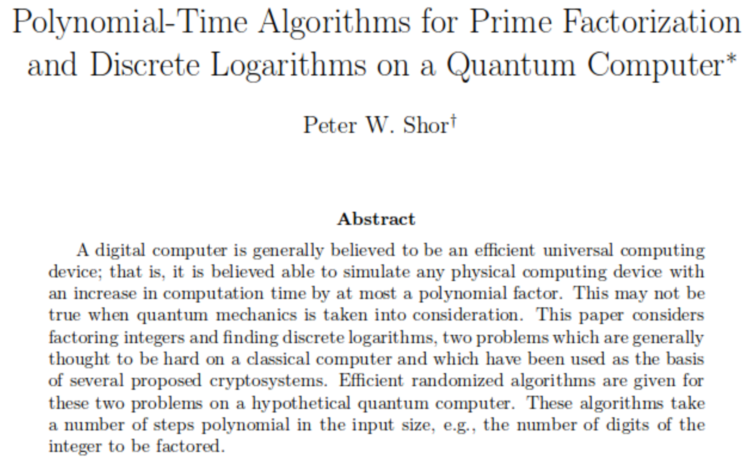
Given $P, Q \in G$ such that $Q \in \langle P \rangle$, find an integer k such that $kP = Q$.

The Discrete Logarithm Problem

Definition

Given $P, Q \in G$ such that $Q \in \langle P \rangle$, find an integer k such that $kP = Q$.

- DH needs group where DLP is hard
- (EC)DLP-based crypto also for signatures (DSA, ECDSA, EdDSA...)
- Prominent alternative: RSA (based on factoring)

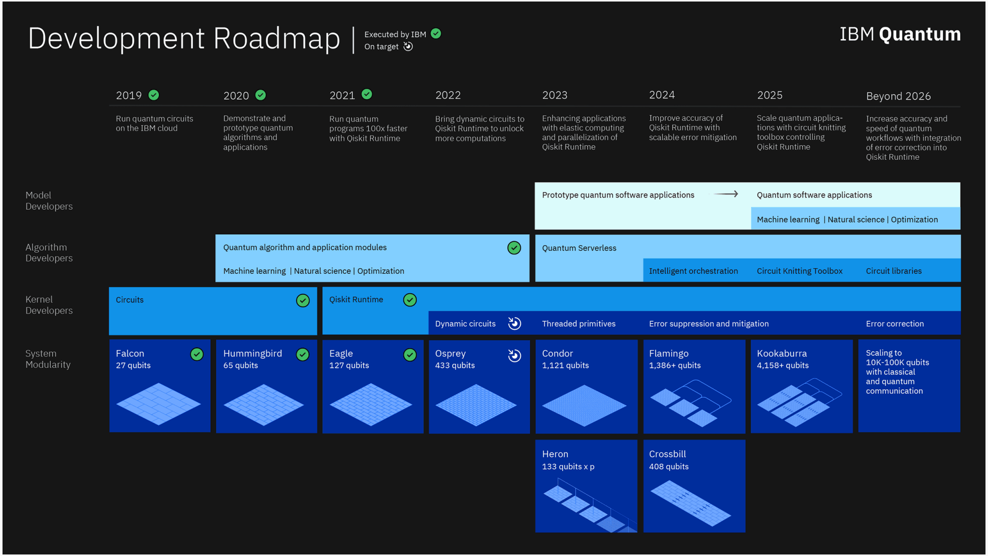


Polynomial-Time Algorithms for Prime Factorization and Discrete Logarithms on a Quantum Computer[*]

Peter W. Shor[†]

Abstract

A digital computer is generally believed to be an efficient universal computing device; that is, it is believed able to simulate any physical computing device with an increase in computation time by at most a polynomial factor. This may not be true when quantum mechanics is taken into consideration. This paper considers factoring integers and finding discrete logarithms, two problems which are generally thought to be hard on a classical computer and which have been used as the basis of several proposed cryptosystems. Efficient randomized algorithms are given for these two problems on a hypothetical quantum computer. These algorithms take a number of steps polynomial in the input size, e.g., the number of digits of the integer to be factored.



Development Roadmap | Executed by IBM ✓ On target ↻

IBM **Quantum**

2019 ✓	Run quantum circuits on the IBM cloud
2020 ✓	Demonstrate and prototype quantum algorithms and applications
2021 ✓	Run quantum programs 100x faster with Qiskit Runtime
2022	Bring dynamic circuits to Qiskit Runtime to unlock more computations
2023	Enhancing applications with elastic computing and parallelization of Qiskit Runtime
2024	Improve accuracy of Qiskit Runtime with scalable error mitigation
2025	Scale quantum applications with circuit knitting toolbox controlling Qiskit Runtime
Beyond 2026	Increase accuracy and speed of quantum workflows with integration of error correction into Qiskit Runtime

Model Developers
- Prototype quantum software applications → Quantum software applications
- Machine learning | Natural science | Optimization

Algorithm Developers
- Quantum algorithm and application modules ✓
- Machine learning | Natural science | Optimization
- Quantum Serverless
- Intelligent orchestration
- Circuit Knitting Toolbox
- Circuit libraries

Kernel Developers
- Circuits ✓
- Qiskit Runtime ✓
- Dynamic circuits ↻
- Threaded primitives
- Error suppression and mitigation
- Error correction

System Modularity
- Falcon 27 qubits ✓
- Hummingbird 65 qubits ✓
- Eagle 127 qubits ✓
- Osprey 433 qubits ↻
- Condor 1,121 qubits
- Flamingo 1,386+ qubits
- Kookaburra 4,158+ qubits
- Scaling to 10K-100K qubits with classical and quantum communication
- Heron 133 qubits x p
- Crossbill 408 qubits

See `https://www.ibm.com/quantum/blog/ibm-quantum-roadmap-2025`

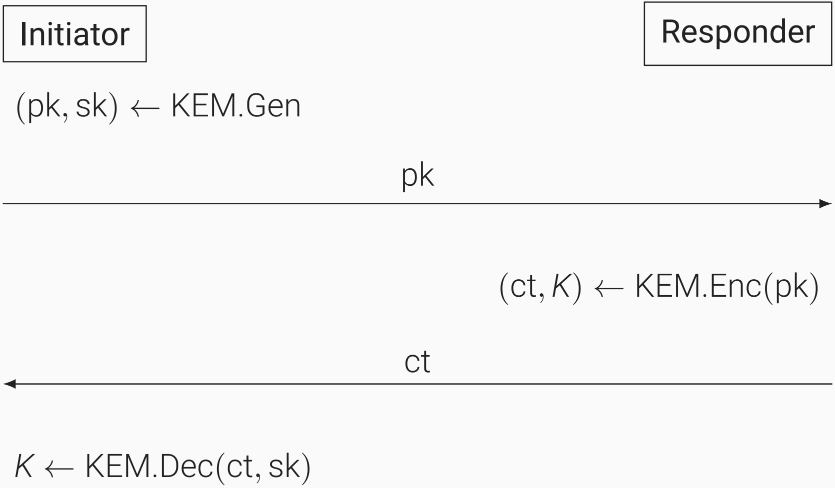
[Back to our demo]

POST-QUANTUM KEY EXCHANGE

A NEW HOPE

ERDEM ALKIM
LÉO DUCAS
THOMAS PÖPPELMANN
PETER SCHWABE

Key Encapsulation Mechanisms (KEMs)



Initiator

Responder

$(\mathrm{pk}, \mathrm{sk}) \leftarrow \mathrm{KEM.Gen}$

$\xrightarrow{\mathrm{pk}}$

$(\mathrm{ct}, K) \leftarrow \mathrm{KEM.Enc}(\mathrm{pk})$

$\xleftarrow{\mathrm{ct}}$

$K \leftarrow \mathrm{KEM.Dec}(\mathrm{ct}, \mathrm{sk})$

Learning with errors (LWE)

- Given uniform $\mathbf{A} \in \mathbb{Z}_q^{k \times \ell}$
- Given “noise distribution” χ
- Given samples $\mathbf{A}\mathbf{s} + \mathbf{e}$, with $\mathbf{e} \leftarrow \chi$

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- Search version: find $\mathbf{s}$
- Decision version: distinguish from uniform random

Ring Learning with errors (RLWE)

- Given uniform $\mathbf{a} \in \mathcal{R}_q = \mathbb{Z}_q[X]/(X^n + 1)$
- Given “noise distribution” χ
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Ring Learning with errors (RLWE)

- Given uniform $\mathbf{a} \in \mathcal{R}_q = \mathbb{Z}_q[X]/(X^n + 1)$
- Given “noise distribution” χ
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- Search version: find $\mathbf{s}$
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How to build a KEM?

Alice (server)		Bob (client)
$\mathbf{s}, \mathbf{e} \xleftarrow{\$} \chi$		$\mathbf{s}', \mathbf{e}' \xleftarrow{\$} \chi$
$\mathbf{b} \leftarrow \mathbf{a}\mathbf{s} + \mathbf{e}$	$\xrightarrow{\mathbf{b}}$	$\mathbf{u} \leftarrow \mathbf{a}\mathbf{s}' + \mathbf{e}'$
	$\xleftarrow{\mathbf{u}}$	

$$\begin{array}{llll} \text{Alice has} & \mathbf{v} & = \mathbf{u}\mathbf{s} & = \mathbf{a}\mathbf{s}\mathbf{s}' + \mathbf{e}'\mathbf{s} \\ \text{Bob has} & \mathbf{v}' & = \mathbf{b}\mathbf{s}' & = \mathbf{a}\mathbf{s}\mathbf{s}' + \mathbf{e}\mathbf{s}' \end{array}$$

- Secret and noise polynomials $\mathbf{s}, \mathbf{s}', \mathbf{e}, \mathbf{e}'$ are small
- $\mathbf{v}$ and $\mathbf{v}'$ are *approximately* the same

How to build a KEM, part 2

Alice		Bob
$\mathbf{s}, \mathbf{e} \xleftarrow{\$} \chi$		$\mathbf{s}', \mathbf{e}' \quad \xleftarrow{\$} \chi$
$\mathbf{b} \leftarrow \mathbf{as} + \mathbf{e}$	$\xrightarrow{(\mathbf{b} \qquad)}$	
		$\mathbf{u} \leftarrow \mathbf{as}' + \mathbf{e}'$
		$\mathbf{v} \leftarrow \mathbf{bs}'$
$\mathbf{v}' \leftarrow \mathbf{us}$	$\xleftarrow{(\mathbf{u} \quad)}$	

How to build a KEM, part 2

Alice		Bob
$\mathit{seed} \xleftarrow{\$} \{0,1\}^{256}$		
$\mathbf{a} \leftarrow \mathsf{Parse}(\mathsf{XOF}(\mathit{seed}))$		
$\mathbf{s}, \mathbf{e} \xleftarrow{\$} \chi$		$\mathbf{s}', \mathbf{e}' \xleftarrow{\$} \chi$
$\mathbf{b} \leftarrow \mathbf{a}\mathbf{s} + \mathbf{e}$	$\xrightarrow{(\mathbf{b}, \mathit{seed})}$	$\mathbf{a} \leftarrow \mathsf{Parse}(\mathsf{XOF}(\mathit{seed}))$
		$\mathbf{u} \leftarrow \mathbf{a}\mathbf{s}' + \mathbf{e}'$
		$\mathbf{v} \leftarrow \mathbf{b}\mathbf{s}'$
$\mathbf{v}' \leftarrow \mathbf{u}\mathbf{s}$	$\xleftarrow{(\mathbf{u}\quad)}$	

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		$\mathbf{u} \leftarrow \mathbf{a}\mathbf{s}' + \mathbf{e}'$
		$\mathbf{v} \leftarrow \mathbf{b}\mathbf{s}'$
		$k \xleftarrow{\$} \{0,1\}^{n}$
		$\mathbf{k} \leftarrow \mathsf{Encode}(k)$
$\mathbf{v}' \leftarrow \mathbf{u}\mathbf{s}$	$\xleftarrow{(\mathbf{u}, \mathbf{c})}$	$\mathbf{c} \leftarrow \mathbf{v} + \mathbf{k}$

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$\mathbf{v}' \leftarrow \mathbf{us}$	$\xleftarrow{(\mathbf{u}, \mathbf{c})}$	$\mathbf{c} \leftarrow \mathbf{v} + \mathbf{k}$
$\mathbf{k}' \leftarrow \mathbf{c} - \mathbf{v}'$		$\mu \leftarrow \mathrm{Extract}(\mathbf{k})$
$\mu \leftarrow \mathrm{Extract}(\mathbf{k}')$		

How to build a KEM, part 2

Alice		Bob
$\mathit{seed} \overset{\$}{\leftarrow} \{0,1\}^{256}$		
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$\mathbf{b} \leftarrow \mathbf{as} + \mathbf{e}$	$\xrightarrow{(\mathbf{b}, \mathit{seed})}$	$\mathbf{a} \leftarrow \mathrm{Parse}(\mathrm{XOF}(\mathit{seed}))$
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		$\mathbf{k} \leftarrow \mathrm{Encode}(k)$
$\mathbf{v}' \leftarrow \mathbf{us}$	$\xleftarrow{(\mathbf{u}, \mathbf{c})}$	$\mathbf{c} \leftarrow \mathbf{v} + \mathbf{k}$
$\mathbf{k}' \leftarrow \mathbf{c} - \mathbf{v}'$		$\mu \leftarrow \mathrm{Extract}(\mathbf{k})$
$\mu \leftarrow \mathrm{Extract}(\mathbf{k}')$		

Encryption scheme by Lyubashevsky, Peikert, Regev. Eurocrypt 2010.

Encode and Extract

- Encoding in LPR encryption: map *n* bits to *n* coefficients:
  - A zero bit maps to 0
  - A one bit maps to $q/2$
- Idea: Noise affects low bits of coefficients, put data into high bits

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  - A zero bit maps to 0
  - A one bit maps to $q/2$
- Idea: Noise affects low bits of coefficients, put data into high bits
- Decode: map coefficient into $[-q/2, q/2]$
  - Closer to 0 (i.e., in $[-q/4, q/4]$): set bit to zero
  - Closer to $\pm q/2$: set bit to one

NEWHOPE (USENIX Security 2016)

- Improve IEEE S&P 2015 results by Bos, Costello, Naehrig, Stebila (BCNS)
- Use reconcilation to go from approximate agreement to agreement
    - Originally proposed by Ding (2012)
    - Improvements by Peikert (2014)
    - More improvements in NEWHOPE

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- Centered binomial noise ψ_k (HW(a)$-$HW(b) for k-bit a, b)
- Achieve ≈ 256 bits of post-quantum security according to very conservative analysis
- Higher security, shorter messages, and $> 10\times$ speedup

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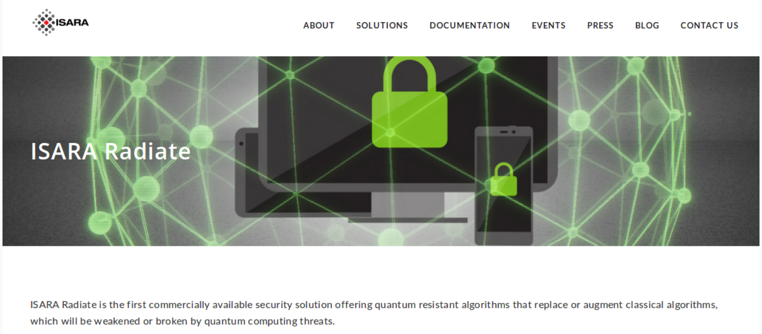
- Multiple implementations

Beyond the paper. . .



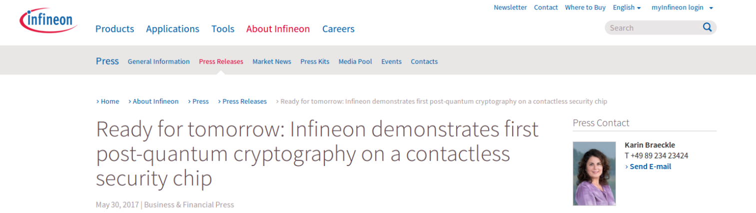
ISARA Radiate

ISARA Radiate is the first commercially available security solution offering quantum resistant algorithms that replace or augment classical algorithms, which will be weakened or broken by quantum computing threats.

“Key Agreement using the ‘NewHope’ lattice-based algorithm detailed in the New Hope paper, and LUKE (Lattice-based Unique Key Exchange), an ISARA speed-optimized version of the NewHope algorithm.”

`https://www.isara.com/isara-radiate/`

Beyond the paper. . .



Ready for tomorrow: Infineon demonstrates first post-quantum cryptography on a contactless security chip

May 30, 2017 | Business & Financial Press

Press Contact

Karin Braeckle
T +49 89 234 23424
› Send E-mail

“The deployed algorithm is a variant of “New Hope”, a quantum-resistant cryptosystem”

`https://www.infineon.com/cms/en/about-infineon/press/press-releases/2017/INFCCS201705-056.html`

Beyond the paper. . .

Google Security Blog

The latest news and insights from Google on security and safety on the Internet

Experimenting with Post-Quantum Cryptography

July 7, 2016

Posted by Matt Braithwaite, Software Engineer

“We’re indebted to Erdem Alkim, Léo Ducas, Thomas Pöppelmann and Peter Schwabe, the researchers who developed “New Hope”, the post-quantum algorithm that we selected for this experiment.”

`https://security.googleblog.com/2016/07/experimenting-with-post-quantum.html`

Also back in 2016: NIST PQC

- National Institute of Standards and Technology
- Public call for PQC proposals, aims at finding schemes for standardization
- Similar to earlier AES and SHA-3 efforts
- Process draft online in August 2016, comments by September 2016
- Call for proposals in December 2016, deadline November 2017

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How it went (so far)

Nov. 2017
69 proposals

Round 1 →

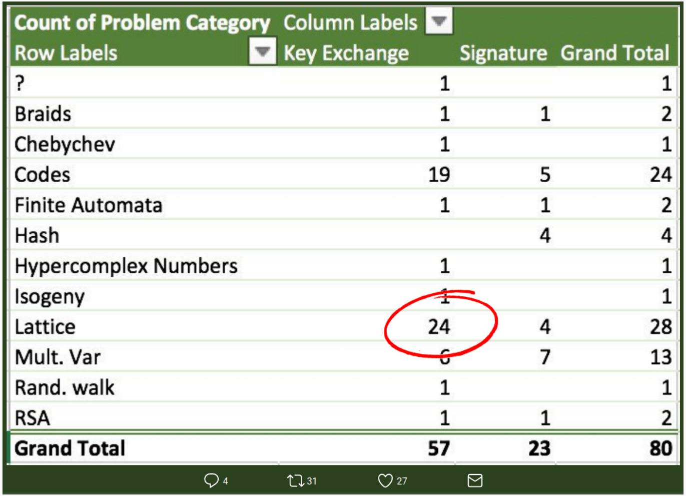
Feb. 2019
26 proposals

Round 2 →

Jul. 2020
7+8 proposals

Round 3 →

Jul. 2022
4 “winners”



Count of Problem Category	Column Labels		
Row Labels	Key Exchange	Signature	Grand Total
?	1		1
Braids	1	1	2
Chebychev	1		1
Codes	19	5	24
Finite Automata	1	1	2
Hash		4	4
Hypercomplex Numbers	1		1
Isogeny	1		1
Lattice	24	4	28
Mult. Var	6	7	13
Rand. walk	1		1
RSA	1	1	2
Grand Total	**57**	**23**	**80**

Overview tweeted by Jacob Alperin-Sheriff on Dec 4, 2017.

Design space 0: The NTRU approach

- Historically first: NTRU
- Use parameters q and $p = 3$

Design space 0: The NTRU approach

- Historically first: NTRU
- Use parameters q and $p = 3$
- **Keygen:**
  - Find $\mathbf{f}, \mathbf{g} \in \mathcal{R}_q$ and $\mathbf{f}_q = \mathbf{f}^{-1} \mod q, \mathbf{f}_p = \mathbf{f}^{-1} \mod p$
  - public key: $\mathbf{h} = p\mathbf{f}_q\mathbf{g}$, secret key: $(\mathbf{f}, \mathbf{f}_p)$

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  - public key: $\mathbf{h} = p\mathbf{f}_q\mathbf{g}$, secret key: $(\mathbf{f}, \mathbf{f}_p)$
- **Encrypt:**
  - Map message m to $\mathbf{m} \in \mathcal{R}_q$ with coefficients in $\{-1, 0, 1\}$
  - Sample random small-coefficient polynomial $\mathbf{r} \in \mathcal{R}_q$
  - Compute ciphertext $\mathbf{e} = \mathbf{r} \cdot \mathbf{h} + \mathbf{m}$

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- Decrypt:
  - Compute $\mathbf{v} = \mathbf{f} \cdot \mathbf{e}$

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  - Compute ciphertext $\mathbf{e} = \mathbf{r} \cdot \mathbf{h} + \mathbf{m}$
- **Decrypt:**
  - Compute $\mathbf{v} = \mathbf{f} \cdot \mathbf{e} = \mathbf{f} \cdot (\mathbf{r} \cdot \mathbf{h} + \mathbf{m})$

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- Use parameters q and $p = 3$
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Design space 0: The NTRU approach

- Historically first: NTRU
- Use parameters q and $p = 3$
- **Keygen:**
    - Find $\mathbf{f}, \mathbf{g} \in \mathcal{R}_q$ and $\mathbf{f}_q = \mathbf{f}^{-1} \mod q, \mathbf{f}_p = \mathbf{f}^{-1} \mod p$
    - public key: $\mathbf{h} = p\mathbf{f}_q\mathbf{g}$, secret key: $(\mathbf{f}, \mathbf{f}_p)$
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    - Map message m to $\mathbf{m} \in \mathcal{R}_q$ with coefficients in $\{-1, 0, 1\}$
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- Advantages/Disadvantages compared to LPR:
  - Asymptotically weaker than Ring-LWE approach
  - Slower keygen, but faster encryption/decryption

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    - q typically either prime or a power of two
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- NewHope and Kyber have fastest (NTT-based) arithmetic

Design space 2: module vs. ring?

- "Traditionally", work directly with elements of $\mathcal{R}_q$ ("Ring-LWE")
- Alternative: Module-LWE (MLWE):
  - Choose smaller n, e.g., $n = 256$ (Kyber, Saber, ThreeBears)
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- MLWE can very easily scale security (change dimension of matrix):
  - Optimize arithmetic in $\mathcal{R}_q$ once
  - Use same optimized $\mathcal{R}_q$ arithmetic for all security levels

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  - **Fixed-weight noise or not?**
    - Fixed-weight noise needs random permutation (sorting)
    - Naive implementations leak secrets through timing
    - Advantage of fixed-weight: easier to bound (or eliminate) decryption failures

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- Solution in NewHope: Choose a fresh $\mathbf{a}$ every time ("Against all authority")
- Server can cache $\mathbf{a}$ for some time (e.g., 1h)
- All NIST PQC candidates ended up using this approach

Design space 5: active security

- Decryption failures are a function of $\mathbf{s}$, $\mathbf{e}$, $\mathbf{s}'$, $\mathbf{e}'$
- Attacker can choose larger secret/noise $\mathbf{e}'$ and $\mathbf{s}'$
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- This is a chosen ciphertext attack (CCA)
- Learn full $\mathbf{s}$ after a few thousand queries
- NEWHOPE never claimed CCA-security!
- This “attack” is completely expected
- Not a problem for ephemeral $\mathbf{s}$

Design space 5: active security (ctd.)

The Fujisaki-Okamoto Transform (idea)

- Build CCA-secure KEM from passively secure encryption scheme
- Make failure probability negligible for honest $\mathbf{s}'$, $\mathbf{e}'$, $\mathbf{e}''$
- Force encapsulator to generate $\mathbf{s}'$, $\mathbf{e}'$, $\mathbf{e}''$ honestly

Design space 5: active security (ctd.)

The Fujisaki-Okamoto Transform

Alice (Server)		Bob (Client)
Gen():		Encaps(pk):
$\mathrm{pk}, \mathrm{sk} \leftarrow \mathrm{KeyGen}()$	$\xrightarrow{\mathrm{pk}}$	$x \leftarrow \{0, \ldots, 255\}^{32}$ $k, \mathrm{coins} \leftarrow \text{SHA3-512}(x)$
	$\xleftarrow{\mathrm{ct}}$	$\mathrm{ct} \leftarrow \mathrm{Encrypt}(\mathrm{pk}, x, \mathrm{coins})$
Decaps((sk, pk), ct): $x' \leftarrow \mathrm{Decrypt}(\mathrm{sk}, \mathrm{ct})$ $k', coins' \leftarrow \text{SHA3-512}(x')$ $\mathrm{ct}' \leftarrow \mathrm{Encrypt}(\mathrm{pk}, x', \mathrm{coins}')$ **verify if** $\mathrm{ct} = \mathrm{ct}'$		

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- Disadvantages:
  - Less robust (will somebody reuse keys?)
  - More options (CCA vs. CPA): easier to make mistakes

Design space 6: CCA transforms

- General Fujisaki-Okamoto principle is the same for most KEMs (exception: NTRU)
- Tweaks to FO transform:
    - Hash public-key into coins: multitarget protection (for non-zero failure probability)

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  - Return special symbol (**`return -1`**): explicit
  - Return $\mathrm{H}(s, C)$ for secret s: implicit
- From round 2, no proposal used explicit rejection
  - Would break some security reduction
  - More robust in practice (return value always `0`)
  - Various recent papers argue **for** explicit rejection

Design space 7: allow failures?

- Can avoid decryption failures entirely (NTRU, NTRU Prime)
- Advantage:
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- Active (CCA) security needs negligible failure probability

Design space 8: error-correcting codes?

- Ring-LWE/LWR schemes work with polynomials of > 256 coefficients
- “Encrypt” messages of > 256 bits
- **Need to encrypt** only 256-bit key
- Question: How do we put those additional bits to use?
- Answer: Use error-correcting code (ECC) to reduce failure probability

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- Answer: Use error-correcting code (ECC) to reduce failure probability
- NewHope: very simple threshold decoding
- LAC, Round5: more advanced ECC
  - Correct more errors, obtain smaller public key and ciphertext
  - More complex to implement, in particular without leaking through timing

KYBER

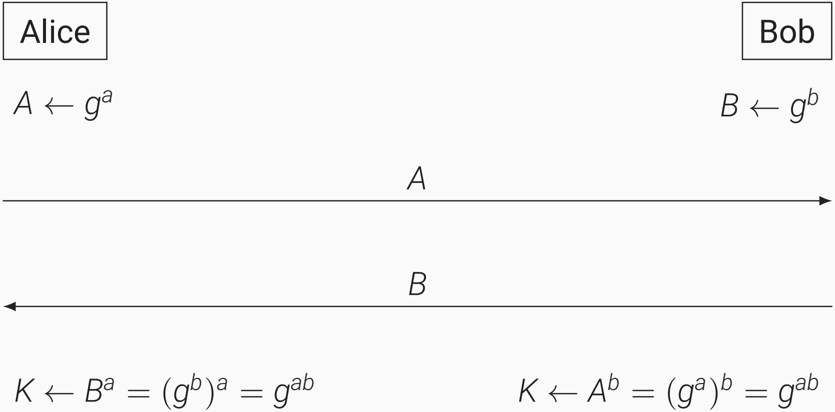
THE KEM

Roberto Avanzi, **Joppe Bos**, **Léo Ducas**, **Eike Kiltz**,
Tancrède Lepoint, **Vadim Lyubashevsky**, **John M. Schanck**,
Peter Schwabe, **Gregor Seiler**, **Damien Stehle**, Jintai Ding

The design of Kyber / ML-KEM

- MLWE version of LPR encryption:
  - Small-dimension vectors and matrices over $\mathcal{R}_q = Zq[X]/(X^{256}+1)$, $q = 3329$
  - Narrow, centered binomial noise with $k = 2$ or $k = 3$
  - Three parameter sets: Kyber-512, Kyber-768, Kyber-1024
- Tweaked FO transform (hash pk into coins and shared key)
- No error-correcting codes; simple encoding of bits into coefficients
- Negligible probability of decryption errors
- All symmetric crypto based on Keccak permutation (SHA-3)

Kyber vs. ECDH: a KEM is not DH!



Alice

Bob

$A \leftarrow g^a$

$B \leftarrow g^b$

A

B

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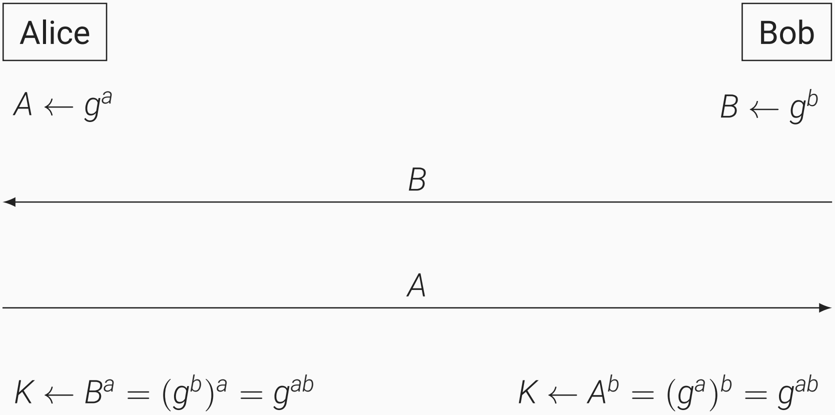

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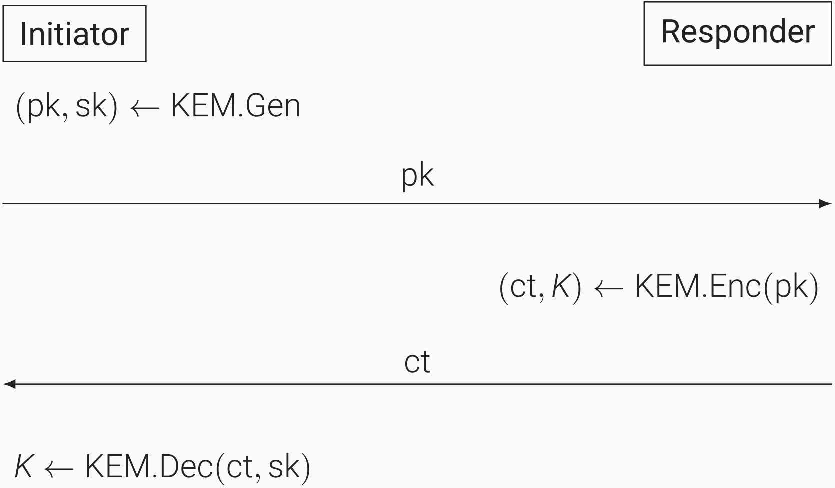

Initiator

Responder

$(\mathsf{pk}, \mathsf{sk}) \leftarrow \mathsf{KEM.Gen}$

pk

$(\mathsf{ct}, K) \leftarrow \mathsf{KEM.Enc}(\mathsf{pk})$

ct

$K \leftarrow \mathsf{KEM.Dec}(\mathsf{ct}, \mathsf{sk})$

Kyber vs. ECDH: performance

X25519 speed

- keygen: 28187 Skylake cycles
- shared: 87942 Skylake cycles

Kyber-768 speed

- keygen: 39750 Skylake cycles
- encaps: 53936 Skylake cycles
- decaps: 42339 Skylake cycles

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X25519 sizes

- public key: 32 bytes

Kyber-768 speed

- keygen: 39750 Skylake cycles
- encaps: 53936 Skylake cycles
- decaps: 42339 Skylake cycles

Kyber-768 sizes

- public key: 1184 bytes
- ciphertext: 1088 bytes

Learn more

- Kyber website: `https://pq-crystals.org/kyber/`
- NIST PQC: `https://csrc.nist.gov/projects/post-quantum-cryptography`
- pqc-forum: `https://groups.google.com/a/list.nist.gov/g/pqc-forum`