Post-quantum cryptography

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August 4, 2016

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"In the past, people have said, maybe it's 50 years away, it's a dream, maybe it'll happen sometime. I used to think it was 50. Now I'm thinking like it's 15 or a little more. It's within reach. It's within our lifetime. It's going to happen."

—Mark Ketchen (IBM), Feb. 2012, about quantum computers

"Whether we can control the quantum states and all of that at the fundamental level has now been proven. The big killer is, at what point do we build a processor big enough thats it's faster than a classical computer?

That means moving away from small scale models to integrated processing devices and prototypes. That's the challenge, and that can be done, we anticipate, within the next decade."

—Michelle Simmons (UNSW), Jan. 2016

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- Complete break of RSA, ElGamal, DSA, Diffie-Hellman
- Complete break of elliptic-curve variants (ECSDA, ECDH, ...)

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- Larger keys, signatures, ciphertexts (for some)
- Security less well understood (for some)
- Additional issues (e.g., stateful hash-based signing)

NIST post-quantum crypto project

- NIST issued a (draft) call for PQC proposals
- Submissions for
  - PQ signatures
  - PQ encryption
  - PQ key agreement
- Submission deadline: November 2017
- Submitters' presentations: Early 2018
- 3–5 years of analysis
- 2 years later: draft standards ready
- See `http://csrc.nist.gov/groups/ST/post-quantum-crypto/`

PQCRYPTO

- Project funded by EU in Horizon 2020.
- Starting date 1 March 2015, runs for 3 years.
- 11 partners from academia and industry, TU/e is coordinator
- Goal: **Design and implement high-security post-quantum PKC**

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- Consequence:

Need post-quantum encryption now!

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- "Perfect Forward Secrecy":
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- *Does not help* against cryptanalytic break
- Attacker breaks (in poly time) each single ephemeral key exchange
- As a consequence, we want
  - **ephemeral key exchange** (to protect against key compromise)
  - **post-quantum security** (to protect against future quantum attacker)

POST-QUANTUM KEY EXCHANGE

A NEW HOPE

ERDEM ALKIM
LÉO DUCAS
THOMAS PÖPPELMANN
PETER SCHWABE

Ring-Learning-with-errors (RLWE)

- Let $\mathcal{R}_q = \mathbb{Z}_q[X]/(X^n + 1)$
- Let χ be an *error distribution* on $\mathcal{R}_q$
- Let $\mathbf{s} \in \mathcal{R}_q$ be secret
- Attacker is given pairs $(\mathbf{a}, \mathbf{as} + \mathbf{e})$ with
  - $\mathbf{a}$ uniformly random from $\mathcal{R}_q$
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- Common choice for χ: discrete Gaussian
- Common optimization for protocols: fix $\mathbf{a}$

A bit of (R)LWE history

- Regev, 2005: Introduce LWE-based encryption
- Lyubashevsky, Peikert, Regev, 2010: Ring-LWE and Ring-LWE encryption
- Ding, Xie, Lin, 2012: Transform to (R)LWE-based key exchange
- Peikert, 2014: Improved RLWE-based key exchange
- Bos, Costello, Naehrig, Stebila, 2015: Instantiate and implement Peikert's KEX in TLS

Peikert's RLWE-based KEM

Parameters: q, n, χ		
KEM.Setup() :		
$\mathbf{a} \xleftarrow{\$} \mathcal{R}_q$		
Alice (server)		Bob (client)
KEM.Gen($\mathbf{a}$) :		KEM.Encaps($\mathbf{a}, \mathbf{b}$) :
$\mathbf{s}, \mathbf{e} \xleftarrow{\$} \chi$		$\mathbf{s}', \mathbf{e}', \mathbf{e}'' \xleftarrow{\$} \chi$
$\mathbf{b} \leftarrow \mathbf{as} + \mathbf{e}$	$\xrightarrow{\mathbf{b}}$	$\mathbf{u} \leftarrow \mathbf{as}' + \mathbf{e}'$
		$\mathbf{v} \leftarrow \mathbf{bs}' + \mathbf{e}''$
		$\bar{\mathbf{v}} \xleftarrow{\$} \mathsf{dbl}(\mathbf{v})$
KEM.Decaps($\mathbf{s}, (\mathbf{u}, \mathbf{v}')$) :	$\xleftarrow{\mathbf{u}, \mathbf{v}'}$	$\mathbf{v}' = \langle \bar{\mathbf{v}} \rangle_2$
$\mu \leftarrow \mathsf{rec}(2\mathbf{us}, \mathbf{v}')$		$\mu \leftarrow \lfloor \bar{\mathbf{v}} \rceil_2$

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Observe: $2\mathbf{us} = 2\mathbf{ass}' + 2\mathbf{e}'\mathbf{s} \approx 2\mathbf{ass}' + 2\mathbf{es}' + 2\mathbf{e}'' \approx \bar{\mathbf{v}}$

BCNS key exchange

- Bos, Costello, Naehrig, Stebila, IEEE S&P 2015:
  - Phrase the KEM as key exchange
  - Instantiate with concrete parameters
  - Integrate with OpenSSL $\rightarrow$ post-quantum TLS key exchange
  - Also: combined ECDH+RLWE key exchange

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- Parameters chosen by BCNS:
  - $\mathcal{R}_q = \mathbb{Z}_q[X]/(X^n + 1)$
  - $n = 1024$
  - $q = 2^{32} - 1$
  - $\chi = D_{\mathbb{Z},\sigma}$
  - $\sigma = 8/\sqrt{2\pi} \approx 3.192$

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  - $\chi = D_{\mathbb{Z},\sigma}$
  - $\sigma = 8/\sqrt{2\pi} \approx 3.192$
- Claimed security level: 128 bits pre-quantum
- Failure probability: $\approx 2^{-131072}$

A new hope

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- Multiple implementations

A new hope – protocol

Parameters: $q = 12289 < 2^{14},\ n = 1024$

Error distribution: ψ_{16}

Alice (server)		Bob (client)
$seed \xleftarrow{\$} \{0,1\}^{256}$		
$\mathbf{a} \leftarrow \mathsf{Parse}(\mathsf{SHAKE\text{-}128}(seed))$		
$\mathbf{s}, \mathbf{e} \xleftarrow{\$} \psi_{16}^n$		$\mathbf{s}', \mathbf{e}', \mathbf{e}'' \xleftarrow{\$} \psi_{16}^n$
$\mathbf{b} \leftarrow \mathbf{as} + \mathbf{e}$	$\xrightarrow{(\mathbf{b}, seed)}$	$\mathbf{a} \leftarrow \mathsf{Parse}(\mathsf{SHAKE\text{-}128}(seed))$
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$\mathbf{v}' \leftarrow \mathbf{us}$	$\xleftarrow{(\mathbf{u}, \mathbf{r})}$	$\mathbf{r} \xleftarrow{\$} \mathsf{HelpRec}(\mathbf{v})$
$k \leftarrow \mathsf{Rec}(\mathbf{v}', \mathbf{r})$		$k \leftarrow \mathsf{Rec}(\mathbf{v}, \mathbf{r})$
$\mu \leftarrow \mathsf{SHA3\text{-}256}(k)$		$\mu \leftarrow \mathsf{SHA3\text{-}256}(k)$

Error reconciliation

- After running the protocol
  - Alice has $\mathbf{x}_A = \mathbf{a}\mathbf{s}\mathbf{s}' + \mathbf{e}'\mathbf{s}$
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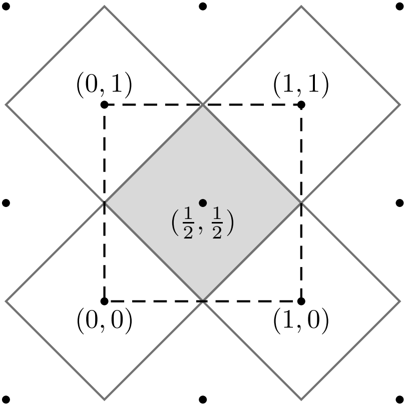
- In the following: 2-dimensional intuition (4-dim. case very similar)
- "Scale" vector $\mathbf{x}$ to $[0, 1)^2$

2D Error reconciliation



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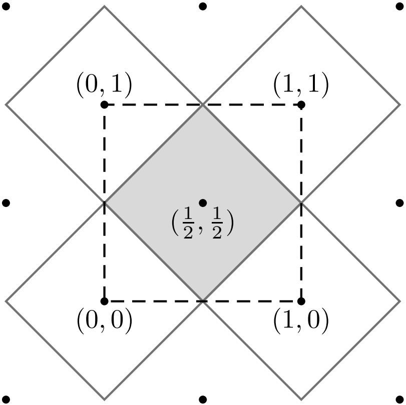

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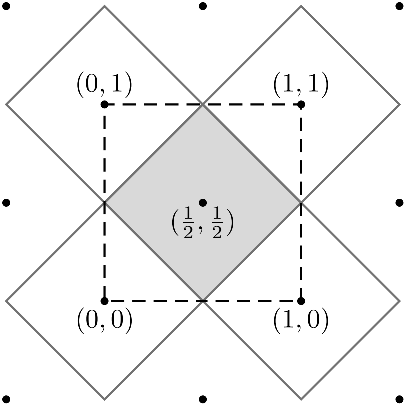

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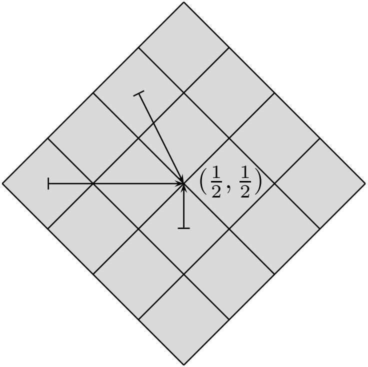
- Reconciliation: Bob sends difference vector from $\mathbf{x}_B$ to center of his Voronoi cell
- Alice adds this difference vector to her vector $\mathbf{x}_A$

Discretization of reconciliation



- Sending difference vector means doubling communcation
- Idea: chop Voronoi cell into 2^{dr} subcells
  - d: dimension (4 for NewHope, 2 in this picture)
  - r: discretization level
- Need to send only rd bits per d coefficients
- NewHope: $r = 2$; hence 256 bytes of reconciliation information

"Blurring the edges"

- This would all work if $\mathbf{x}$ was continuous uniform from $[0, 1)$
- We start with $\mathbf{x} \in \{0, \ldots, q-1\}^2$, q odd
- Odd number of possible values; no way to pick key bit without bias!
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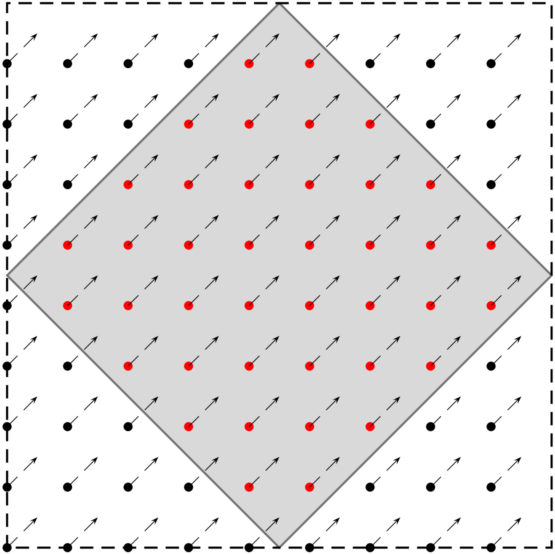

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- Dual attack: find short vector in dual lattice
- Length determines complexity and attacker's advantage ϵ

JarJar

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—*anonymous reviewer*

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- JarJar: instantiation with $n = 512$
- Same $q = 12289$
- Use root lattice D_2 instead of D_4
- Use $k = 24$ for the centered binomial distribution

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JarJar is not recommended for use!

Post-quantum security

Attack	m	b	Known Classical	Known Quantum	Best Plausible
BCNS proposal: $q = 2^{32} - 1$, $n = 1024$, $\sigma = 3.192$					
Primal	1062	296	86	78	61
Dual	1055	296	86	**78**	61
JarJar: $q = 12289$, $n = 512$, $\sigma = \sqrt{12}$					
Primal	623	449	131	119	93
Dual	602	448	131	**118**	92
NewHope: $q = 12289$, $n = 1024$, $\sigma = \sqrt{8}$					
Primal	1100	967	282	256	200
Dual	1099	962	281	**255**	199

- b: Block size for BKZ
- m: Number of used samples

Against all authority

- Remember the optimization of fixed $\mathbf{a}$?
- What if $\mathbf{a}$ is backdoored?
- Parameter-generating authority can break key exchange
- "Solution": Nothing-up-my-sleeves (involves endless discussion!)

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- **Must not reuse keys/noise!**

NTT-based multiplication

- Most costly arithmetic operations: multiplication in $\mathcal{R}_q$
- Idea behind selecting n and q: fast negacyclic number-theoretic transform (NTT)
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- NTT takes $\frac{n}{2}\log(n)$ "butterfly operations"
- Butterflies are one addition, one subtraction, one multiplication by constant

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- AVX2 implementation:
  - Speed up NTT using vectorized `double` arithmetic
  - Use AES-256 for noise sampling
  - Use AVX2 for centered binomial

The protocol revisited

Parameters: $q = 12289 < 2^{14}$, $n = 1024$

Error distribution: ψ_{16}^n

Alice (server)		**Bob (client)**
$seed \xleftarrow{\$} \{0,\ldots,255\}^{32}$		
$\hat{\mathbf{a}} \leftarrow \mathsf{Parse}(\mathsf{SHAKE\text{-}128}(seed))$		
$\mathbf{s}, \mathbf{e} \xleftarrow{\$} \psi_{16}^n$		$\mathbf{s}', \mathbf{e}', \mathbf{e}'' \xleftarrow{\$} \psi_{16}^n$
$\hat{\mathbf{s}} \leftarrow \mathsf{NTT}(\mathbf{s})$		
$\hat{\mathbf{b}} \leftarrow \hat{\mathbf{a}} \circ \hat{\mathbf{s}} + \mathsf{NTT}(\mathbf{e})$	$\xrightarrow[\text{1824 Bytes}]{m_a = \mathsf{encodeA}(seed, \hat{\mathbf{b}})}$	$(\hat{\mathbf{b}}, seed) \leftarrow \mathsf{decodeA}(m_a)$
		$\hat{\mathbf{a}} \leftarrow \mathsf{Parse}(\mathsf{SHAKE\text{-}128}(seed))$
		$\hat{\mathbf{t}} \leftarrow \mathsf{NTT}(\mathbf{s}')$
		$\hat{\mathbf{u}} \leftarrow \hat{\mathbf{a}} \circ \hat{\mathbf{t}} + \mathsf{NTT}(\mathbf{e}')$
		$\mathbf{v} \leftarrow \mathsf{NTT}^{-1}(\hat{\mathbf{b}} \circ \hat{\mathbf{t}}) + \mathbf{e}''$
$(\hat{\mathbf{u}}, \mathbf{r}) \leftarrow \mathsf{decodeB}(m_b)$	$\xleftarrow[\text{2048 Bytes}]{m_b = \mathsf{encodeB}(\hat{\mathbf{u}}, \mathbf{r})}$	$\mathbf{r} \xleftarrow{\$} \mathsf{HelpRec}(\mathbf{v})$
$\mathbf{v}' \leftarrow \mathsf{NTT}^{-1}(\hat{\mathbf{u}} \circ \hat{\mathbf{s}})$		$k \leftarrow \mathsf{Rec}(\mathbf{v}, \mathbf{r})$
$k \leftarrow \mathsf{Rec}(\mathbf{v}', \mathbf{r})$		$\mu \leftarrow \mathsf{SHA3\text{-}256}(k)$
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Key generation (server)	$\approx 2\,477\,958$	$258\,246$ ($258\,965$)	$88\,920$ ($89\,079$)
Key gen + shared key (client)	$\approx 3\,995\,977$	$384\,994$ ($385\,146$)	$110\,986$ ($111\,169$)
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- Benchmarks on one core of an Intel i7-4770K (Haswell)
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- Also optimize other building blocks of NewHope

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- Comparison to HECC: ≈ 2.63 cycles on Kummer surface on M0

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NewHope in TLS

- Google is running a post-quantum experiment
- Combination of NewHope and X25519 (called CECPQ1)
- Some connections from Chrome Canary to some Google services
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- Plan: Merge these proposals

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- How about NTRU Prime?
  - Paper by Bernstein, Chuengsatiansup, Lange, van Vredendaal
  - See `http://eprint.iacr.org/2016/461`
  - Useful for ephemeral key exchange?

NewHope online

Paper:	https://cryptojedi.org/papers/#newhope
Software:	https://cryptojedi.org/crypto/#newhope
ARM Paper:	https://cryptojedi.org/papers/#newhopearm
ARM software:	https://github.com/newhopearm/newhopearm.git
Newhope in Go:	https://github.com/Yawning/newhope (by Yawning Angel)
Newhope in Rust:	https://code.ciph.re/isis/newhopers (by Isis Lovecruft)
Newhope in Java:	https://github.com/rweather/newhope-java (by Rhys Weatherley)
Newhope in Erlang:	https://github.com/ahf/luke (by Alexander Færøy)